REINFORCEMENT LEARNING WITH MODEL DRIVEN APPROACH IN GAMIFICATION MODELS OF GIN RUMMY

Anil Kumar S,
M.Tech (Software Engineering) Final year,
Department of Computer Science,
Cochin University of Science and Technology (CUSAT),
Cochin, India – 682 022

Muralidharan K B,
Assistant Professor,
Department of Computer Science,
Cochin University of Science and Technology (CUSAT),
Cochin, India – 682 022

Abstract: Adaptive Learning Systems, as in Robotics, require taking most optimized decisions dynamically, based on the feedback from their environment. Also they have to enhance their knowledge by experience and represent the knowledge in a reusable form. Developing the actual models of such self learning systems is quite complex, expensive and time consuming. Concepts from certain gaming and gamification models exhibit resemblance to solution models of several real world problems. A model driven approach is helpful for formulating reusable meta-models from simpler and affordable gamification models which exhibit almost similar characteristics as in the actual solution models. Experimental learning with such meta-models will help to formulate the strategy for optimizing the actual solution models as well. This study explores the scope and challenges of applying model driven approach while architecting Reinforcement Learning Solutions using meta-models from Gaming and Gamification. Simple analogies are illustrated with Gamification Models derived from gaming meta-models based on Gin Rummy Game.

Keywords: Reinforcement Learning, Model Driven Architecture, Meta-models, Gamification,Clustering,Gin Rummy

I. INTRODUCTION

Solution models for most of the real world problems are much expensive and complex to develop. But affordable meta-models can be derived from various gaming models. These meta-models can be the basement for building actual architectural models for software solutions. Such a Model Driven Approach [1] facilitates reuse of architectural patterns through one or several layers of meta- models.

Gamification[2] is the upcoming trend of applying gaming concepts to formulate simpler and interesting solution models for real world problems. Often the gaming concepts in the pure form may not be suitable to apply in the real world models. So, meta-models [1] and patterns [3] for the actual solution models are formulated from simpler concepts in games.

In Adaptive Learning Systems, like Robots, solution strategies are formulated by experiencing the real environment. Decision making process has to be carried out according to the feedback from the environment. Base models of the solutions can be formulated by learning tactics in gaming and gamification models. Such solution models can be used for engineering the complex real world problems.

While implementing such solutions, special learning techniques like Reinforcement Learning [4] are to be employed, rather than using mathematical and statistical models alone.

II. GAMING MODELS FOR META MODELLING

Most of the tactical concepts in games are derived from situations in real life problems. For example, chess is a lighter simulated version of wars. By applying RL approach, one can actually experience such an architectural process in gaming. Best way for this is to try for architecting a new game. Formulation of a totally new game is highly challenging. When asked to do so, innovative brains will automatically seek for problem patterns from their previous experiences in real world or other gaming models. Mostly, a new gaming concept is getting born by combining ideas from such meta-models. Such ideas can originate from life situations experienced by oneself or from the events observed among others, or even from existing games.

Even if one is trying to derive new games by modifying or combining concepts in existing games, there also a model driven approach can be observed. For example, the Olympics game event known as 'Synchronized Swimming' is a model derived from two gaming meta-models, i.e. Swimming and Gymnastics.

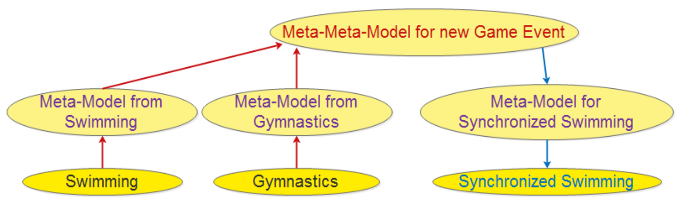


Figure 1. Meta-modeling concept in Model Driven Approach

Popular games, as everyone knows, pay highest care for enhancing the user experience by adding cosmetic elements with the serious concepts from the real life situations. Thus the games will become simpler and interesting for the crowd.

In fact, game modeling needs effective utilization of skills from multiple areas of study like arts, science, technology, mathematics, literature, psychology and so on. The next few sections discuss the various aspects (scope, challenges etc) in deriving gamification models based on the concepts from Gin Rummy [5].

III. GIN RUMMY GAME

It is a simple card game popular in computers and mobile devices [6]. It is a lightened version of popular card game Rummy. Gin Rummy is a game for two players at a time, ie it is a two agent learning problem. Basic rules and concepts of the game used for this study are summarized as follows.

A. *The Deck*

One standard deck of 52 cards (13 cards each in the four groups Spades[♠], Diamonds[♦], Clubs[♣] and Hearts[♥]) is used. Since Gin Rummy do not allow substitutes, Joker cards are not used. Cards in each suit are ranked, from low to high

Table 1. Cards used and their cost points in Gin Rummy

SL No	Standard Deck of (13 x 4 = 52) cards	
	Card	**Cost Points**
1	Ace	1
2-10	2,3,4,5,6,7,8,9,10	Numeric value associated with
11	Jack	10
12	Queen	10
13	King	10

B. *The Deal*

After shuffling the deck, both the players are given 10 cards each, ie the alternative ones on the top of the deck as done in usual card games. The twenty first card is turned face up and kept as the initial card in the Discard pile on the table. The remainder of the deck is placed face down on the table near the discard pile to from the stock.

As in most of other card games, the players are permitted to see only the cards in their hand, and those on the top of the Discard pile. The players can sort or rearrange the cards in their hand for the convenience of forming the set of related cards as described in the following paragraphs.

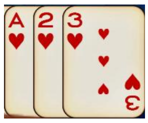
C. *Objective of the Game*

The Objective of the game is to form sets of related cards. As the point values of the unmatched cards will be treated as deadwoods at the end of the game and result it penalty, the aim of each player is to reduce the total value points of the unmatched cards.., i.e. to make it lesser than the opponent.

D. *Formulation of Sets*

As in Rummy game, two type of sets can be formulated in Gin Rummy.

- A ***Sequence Set*** (also known as a *run, or a sequence*) is a cluster formed with three or more cards of the same suit in continuous order. For Example



or

- An ***Ordinary Set*** (also known as group or even in the generic name *set*) is the cluster formed with three or four cards of the same rank; but (obviously). in different suits. For Example

or

Each card is permitted to be the part of only one set. When there are overlapping clusters, i.e. a same card is the part of two different sets, the player can go for the more beneficial option, i.e. to include the common card to form group which minimises the total value points of unmatched cards (termed as ***dead-wood*** at the end of the game).

Here in the following situation, the player is having two options to formulate a set using Diamond Six; ie a Sequence set(4-5-6) and an Ordinary set (6-6-6); but can choose only one among them.

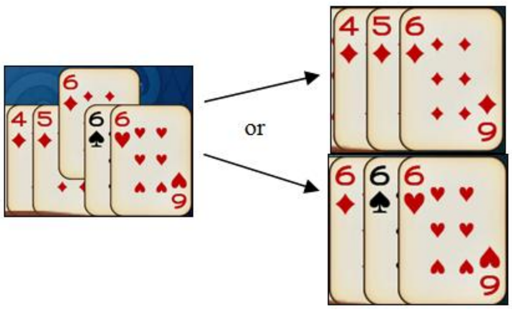


Figure 2. Problem of choosing the optimal selection when a card can be part of two possible sets

If the player is opting to form the set with three Sixes (6-6-6), the total contribution towards the dead wood is only [4+5=9]. Otherwise, If opting to form the sequence set, the contribution to the deadwood will become [6+6=12]. Hence, here the better option for the player is to go ahead with forming the set of three sixes(6-6-6) which will help to minimise the deadwood. However in usual Rummy game, the decision has to depend on other rules too, as two sequence sets are compulsory (In Gin Rummy, sequence sets are not compulsory for a knock). Practical utilization of this constraint as a tactic is described in the subsequent pages.(Maintaining Overlapping clusters among deadwood cards).

Unlike in usual Rummy, an Ace is given the lowest rank only. In usual Rummy game, Ace is having both the lowest and highest ranks.

Table 2. Comparison of rules in usual Rummy and Gin Rummy for validating for Sequence Sets formed using Ace

Set of Cards	Rummy	Gin Rummy
	Valid	Not Valid
	Not Valid	Not Valid
	Valid	Valid

E. *The Play*

The normal turn of each player comes in rotation. A normal turn consists of two parts.

- **Draw :** Player can take a card either from the top of stack or from the top of the discard pile.
    - While drawing from the discarding pile, as the card's face is up, the player can know the card before taking it; Also the opponent can see that card. But the player gets the advantage that to verify the card in advance whether it is useful or not.
    - However, while drawing a card from the stock, as the card's face is down, the player cannot check the card's usefulness until getting committed to take it. However, after taking the card the player can see it; without showing it to the opponent.
- **Discard :** Drawing a card results in excess of a card. So after each draw, the player has to discard a card to the discard pile. If the card is just drawn from the top of the discard pile, the player should discard a different card.

The first turn, ie while drawing the first card, the turn is decided through the mutual agreement between the dealer and non-dealer players.

F. *Knocking*

The players can knock for finishing a game when the turn comes, ie after drawing a card. Instead of discarding the excess card, the player can knock, if confident enough to take the expectation that the opponent is having more deadwood value. Knocking (claim for a win) can be done if the total of the deadwood cards is less than or equal to 10.

G. *Scoring*

Scoring is done based on the sum of point values associated with deadwood cards. Knocking player gets positive score if the total value of deadwood is less than that of the opponent. If the expectation is wrong, ie the opponent is having exactly same or lesser total for deadwood, the knocking player will get an additional penalty (The opponent gets more points). Making a 'Gin' (a finish with zero deadwood) will help the player to earn bonus points.

H. *Multi-Agent Game*

The multi agent game version of Gin Rummy simulates the one followed other similar 2-opponent games, ie winners of the preliminary rounds will fight each other in the subsequent level, gradually reducing the number of players until reaching the one and only winner. So the Multi-Agent gaming models of Gin Rummy can be implemented using a parallel or distributive computing architecture; so the corresponding gamification models too.

IV. LEARNING TACTICS OF GIN RUMMY GAME THROUGH REINFORCEMENT LEARNING

Repeatedly playing the game will help a learner agent to identify some tactics which will help to increase the probability for winning in future games.

Useful (harmful also) tactics can be identified through playing the game repeatedly with a Reinforcement Learning Approach. Usually, most of us play the game without any experimental intentions. Even then, the players unknowingly learn all such tactics through experiencing the problem (Reinforcement Learning automatically happens). A few examples of such tactics are explained below based on a real time instance while playing the Gin Rummy game.

A. A Real Time Situation from Gin Rummy Game

Figure-3 : A situation from Gin Rummy Game while Playing with the Mobile Application

Here the player has just drawn a card, and to discard the least useful one. The usefulness of a card is to be assessed based on how far the card (if maintained) is helpful for reaching to victory at earliest (or reducing the deadwood) or to block the opponent's victory. Some of such tactics are described below.

A. Tactic 1: Maintain overlapping Clusters among deadwood cards

For example, in the above situation, the player is applying the tactics of maintaining overlapping clusters among the deadwood so that the arrival of a related card will help to formulate sets in multiple ways. This tactic provides the freedom of making set in most optimal manner.

In the figure, an expected diamond 7 is related to more than one group. So the player can form successful sets in more than one way if getting a diamond 7.

B. Tactic 2 : Maintain multiple options open

Another tactic used by this player is that of keeping multiple options for complementary cards for deadwood cards. i.e. a set of two related deadwood cards are able to form a set on arrival of one among several cards. Here the group of spade 8 and 7 are keeping both sides open. i.e. either a spade 9 or spade 6 will help to formulate a sequence set. On the contrary to this, the spade Ace and spade 2 compulsorily need a single card, ie spade 3 to form a set.

C. Tactic 3: Selection of card to discard

Another tactic to be applied is in discarding excess card from the deadwood. During the beginning stages of the game or if being an optimist, the player may choose a risky option of leaving the diamond 3, (i.e. the card with one and only one option [diamond 4] for the completion, and with the logic that arrival of [diamond 4] can formulate a complete sequence [4-5-6] even if the [diamond 3] is not preserved). If the game is progressing to the end or if the player is pessimistic, the better option is to leave the spade 8, which will reduce the risk of total deadwood points by 5, even if an opponent will claim for the finish of the game.

V. THEORETICAL CONCEPTS

The core tactics in this game are closely related to Clustering problems. So gamification models use the theoretic concepts from various branches of study like Machine Learning [7], Agent Based Computing, Meta-Modelling, Operations Research and Software Engineering to name a few.

A. Hierarchical Clustering

The gaming experience in Gin Rummy is very similar to the Hierarchical Clustering Problems. To arrange the cards in hand, the player applies Hierarchical Agglomerative Clustering (HAC); ie in the beginning of the game, all the 10 cards can be considered as in separate clusters and then the clusters are updated by merging related clusters. Also, for rearranging the cards, the player first applies the Hierarchical Division Clustering (HDC) to separate the cards which are already clustered and then apply the HAC again to redefine the clusters.

B. Multi-Agent Model for Distributive Computing

Multi-Agent models can support distributive computing if the players can carry out the learning or testing processes independent to each other. In Gin Rummy, different gaming instances ([player1 vs player2], [player3 vs player4], ...) are independent and can be carried out in different locations and even in a parallel manner. Advantage of applying this concept can be experienced specially in problems exhibiting Associative Law among the sub problems or components. See the example of LCM Calculator (in subsequent section) when used to process large number of inputs.

C. Meta-Modelling For Enhanced and Variant Application Models

Most of the gaming models simulate characteristics of real world application models. However, usually the game models will contain added flavours of exaggerations, simplifications, fun and entertainment. So, in most of the cases, the game models cannot be simulated as it is in the derived application model. The complex practical applications may need simulation of concepts from multiple games. So a model driven approach can be applied for formulating meta-models of gaming models from which application models can be derived. Also, the modelling and meta-modelling may have to be done in several layers of model driven architecture.

VI. POTENTIAL AREAS OF PROBLEM SOLVING - EXAMPLES

There are several situations in real world, which simulates the situations in Gin Rummy with or without some variations. In most of such situations, the stakeholders need to play some trial and error efforts for planning as well as formulating tactics to optimize their work. The Gamification Models based on Gin Rummy can be used in such situations. Some potential areas are mentioned below.

A. Component –Driven Software Development

Combining components of perfect blend can make fully useful features; i.e. like the completed sets in Gin Rummy. The pending back logs can be represented using simulations of the deadwood cards. At each state (each phase of the project) the players (project team members) can analyse the overall cost of the deadwoods, ie the pending works.

In software industry, there is variation from the base model. Here the same component can be reused in multiple project features, while in Gin Rummy one card can be included in only one group.

B. Resource Management

Resource allocation activities with the aim of maximising the profits and the minimising Loss need complex optimisation on tactics. While adding and releasing resources, the 'value points' for decision making change..., as experienced while dynamically calculating the deadwood score during the drawing and discarding activities in Gin Rummy.

C. Team Building

The gaming concept of Gin Rummy can be applied in formulation of Team building activities by clustering the employees with similar or complementary skill sets. Often the skill sets of employees can have an overlapping nature and so the gamification model shall be able to handle overlapping clusters.

D. Collaborating Experts

For projects and products, formulating collaborations of experts from various fields is required. While nominating expert members into the different slots in a team, a lot of conditional factors need to be considered and optimised..., as somewhat similar to managing the cards in Gin Rummy.

E. Idea formation for Media :: Cartoons and News Based Programmes

Clustering of related and contradictory concepts from the existing ideas, with the help of Reinforcement Learning and Model Driven based approaches, can lead to formation of innovative ideas for new media products by combining elements from existing stuff.

F. Other Potential Areas in Real world

Analogies of strategies in this game can be seen in several Business Process models (Budget Optimization, Resource allocation etc for example) and Software Engineering (Product Line Engineering, Model driven Architecture, Patterns based Software Development etc for example).

Even in Creative Writing, the concepts from Gin Rummy can be helpful for writers to formulate new story concepts by randomly clustering the existing ideas. In Multi-Lingual Computing, the Clustering concepts from Gin Rummy model can be applied for generating meaningful combinations of letters to form words, of words to form sentences, of sentences to form stories and so on.

The above list of domain areas will not end, if observing the real world entities with an analytical mind. An experimental study of formulating a simple Gamification Model (based on meta-models of Gin Rummy) to find the HCF and LCM of various numbers is illustrated below.

VII. GAMIFICATION MODEL FOR FINDING HCF AND LCM

From the above examples one can identify that Gamification Models for various industrial domains can become quite complex with lot of variations and additions in the base gaming model. A very common example from simple mathematics is illustrated below. ie simulating the concepts of Gin Rummy in finding Least Common Multiple(LCM) and Highest Common Factor (HCF).

The method of using gamification meta-model of Gin Rummy may not be the best approach for solving the LCM problem. (There are more efficient ways for automated solutions through recursive algorithms which converge to solution in a more efficient manner). However, the following example of LCM and HCF models based on Gin Rummy is useful for experiencing the process of deriving a Gamification model, in the context of Reinforcement Learning and Model Driven Approach, which can be used as meta-model for many complex problems having similar nature.

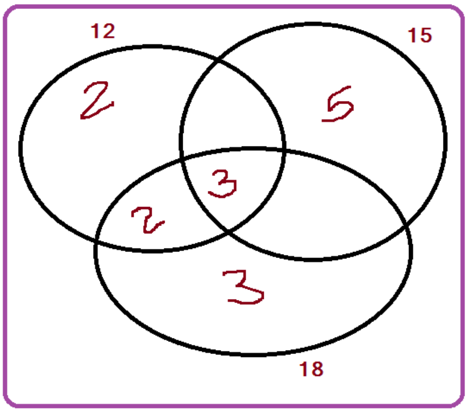


Figure 4. A clustering approach in Gamification model for finding LCM from the Factors of three numbers (12,15 and 18).

A. Meta Models from Gaming and Mathematics

It seems to be silly to solve simple LCM calculation process is implemented with this complex concept. Further, a gaming model for finding LCM with playing cards may seem to be silly and childish; but this can be a very good example model to learn how to apply Gamification concepts and Meta-Modelling in Problem Solving, in the context of Reinforcement Learning. Most of the human understands and solves the problem in a similar manner.

The most popular techniques used for finding LCM and HCF are Factorisation Method and Division Method. In both methods, the answer is derived based on the prime factors. In the proposed model, features of both the methods are required. Gin Rummy Game model cannot be used as it is, as some of its features are to be modified. So the proposed model can be derived from the meta models of the gaming model of Gin Rummy and mathematical model of calculating HCF and LCM.

B. Variations from base models

Each Number is assigned to distinct Player Agents. So it is a multi-agent problem. So in a simple software model (without parallel or distributive processing) the number of player agents is same as the number of Input values whose LCM and HCF are to be calculated.

Unlike Gin Rummy, this special model permits each player to perform their learning (Drawing and Discarding activities) activity independently. So the player agents are given separate deck of cards. Each deck contains cards of unique suit. The pile of cards are marked with prime numbers in the range of largest possible input. Each player uses its own deck to play.

The Gin Rummy Based LCM/HCF Calculator has two Components for its Processing Phase. The first Component, the one used for generating the prime factors, is derived from Factorisation Method. It is a simulation of the Drawing and Discarding Activity in Gin Rummy. However, in this gamification model, discarding activity is not compulsory after each draw; i.e. if the Agent finds that a card drawn from the pile is useful, it is kept in hand (no cards will be discarded), otherwise it is simply discarded. In the gaming model, these two concepts are totally opposite. This part of the process is distributable, i.e. the agents need not be taken place in the same location. The Second Component of the Processing Phase is used for consolidation of the outputs of the first component. Here entire values are needed together in a location as in Division Method, and the process is less distributable.

C. The Process Model

The steps involved for implementing above model are described as follows using a simple example model. Two copies each of the three types of cards (labelled with three prime numbers involved in the above model ; 2, 3 nd 5) are used, for the purpose of explaining the model. (In an actual situation more cards are required to form a generic model which work irrespective of the input values , as described in the subsequent sections). The following model is a simplest one for processing three numbers 12,15 and 18.

Step- 0 :: Intiallising the decks for each player with prime number cards

The deck for each Learner Agent can be as follows (may be ordered or shuffled based on the optimization requirements). Enough copies of same card are to be used. Here in this simple example, two copies of each card are included)

Table 3. Deck of cards for each player in the simple Gamification model for finding LCM and HCF of three numbers (12,15 and 18).

Player	Number being processed by the player	Pile of Cards representing the Deck for each player. (with multiple copies of prime numbers in the range..different suits for each player)					
1	12	2♦	2♦	3♦	3♦	5♦	5♦
2	15	2♣	2♣	3♣	3♣	5♣	5♣
3	18	2♥	2♥	3♥	3♥	5♥	5♥

Step- 1 :: Generating Prime Factors by each Player

After this step, each player will be with the useful cards in their hands

Table 4. Cards in the Hands and Discard piles of each player.

Player	Cards In Hands				Cards in Discard piles		
1 (12)	2♦	2♦	3♦		3♦	5♦	5♦

2 (15)	3♣ 5♣	2♣ 2♣ 3♣ 5♣
3 (18)	2♥ 3♥ 3♥	2♥ 5♥ 5♥

The cards in the discard piles are simply ignored and those in hands of each player are given as the input for the next phase (consolidation).

Step- 2 :: Consolidating the cards in hands of all the players

In this step the cards in the hands of all the players are brought together. The cards in the discard piles are simply ignored. After performing this step, the collection of short listed cards will be as follows.

Table 5. Short listed cards are consolidated.

2♦	2♦	3♦	3♣	5♣	2♥	3♥	3♥

This consolidated collection is the input for next step (clustering).

Step- 3 :: Clustering the cards to form Full and Partial Sets

After the consolidation is done, Clustering is applied as in the case of Gin Rummy to form the sets.

Table 6. Clustering the cards based on the rank as done in Gin Rummy (Obviously in this gamification model there are no sequence sets)

Cluster	1	2	3	4	5
Cards	3♦ 3♣ 3♥	2♦ 2♥	2♦	3♥	5♣
Score	3	2	2	3	5

Suppose the number of players is N. Then the maximum number of cards to form a set is also N. Each cluster shall contain the cards of same value; but of distinct suits. The dead wood can be used to form Partial clusters, i.e. may be with [N-1] or [N-2],.... or [1] cards in them.

Step- 4 :: Special Scoring Functions for finding HCF and LCM

As the purpose is different from the Gin Rummy game, the Scoring function needs to be changed. For finding the HCF, only the Perfect clusters (only the cluster -1 in the above example) are taken. For LCM, all the clusters (ie perfect ones and partial ones) are taken.

Table 7. Calculation of HCF and LCM

Final Calculation					
Score	3	2	2	3	5
Nature	Perfect	Partial			
HCF	**3**	2	2	3	5
	3 (Product of Scores of Perfect Clusters only)				
LCM	**3**	**2**	**2**	**3**	**5**
	180 (Product of Scores of both Perfect and partial Clusters)				

The result is obtained by multiplying the value represented by elements of the selected clusters.

D. Analysis of the Solution Model

The experiments can be repeated in several epochs so as to formulate the Optimisation strategies. However, in this specific problem, the sizes of the Deck and number of cards can be optimised based the upper and lower limits of the input numbers.

Optimisation 1 :: Limiting the Range of Cards

If the upper limit of incoming numbers is known to be 'M' the prime numbers required to form the deck can be limited in the range from [2] to [M]. So, apart from the playing cards in the standard deck of fifty two cards, user defined cards are required when the M exceeds the limit of 13 or the number of simultaneous players goes beyond 4.

Optimisation 2 :: Limiting the Deck by fixing the number of duplicates required.

Duplicates of same cards are required to process higher numbers as the prime factors will get repeated depending on the logarithmic value. In this model, since the number of duplicates required for each prime number card can be determined in advance, there is no need for maintaining an unlimited source with infinite number of copies of each card.The number of copies for each prime number card required in each deck can be determined using the following formula.

$$D = \text{Truncate}\ (\log_P M)$$

where,

P : The Prime Number which labels the card
D : Number of cards required for P in each deck
M : Maximum Limit of Input. If the decks are initialised after receiving the input value for the player, the value of M can be set as the input value itself.
Log : Function to calculate Logarithmic value of M with the base P
Truncate : Function to find the Integer part of a real number without rounding.

For Example

If P is 2 and M is 70, then

$D = \text{Truncate}(\log_2 70)$

$= \text{Truncate}(6.12928302)$

$= 6.$

So, for finding HCF and LCM of numbers up to 70, there should be 6 copies of Playing card representing the prime number 2. Following table summarises the number of copies of cards required for formulating the suits for LCM and HCF Finder which can process numbers up to 70.

Table 8: Number of cards required for each prime numbers for a model with input values ranging from 2 to 70.

P	$\log_P 70$	D	P	$\log_P 70$	D
2	6.12928302	6	43	1.12955842	1
3	3.86714702	3	47	1.10346295	1
5	2.63973851	2	53	1.07007125	1
7	2.18329466	2	59	1.04192673	1
11	1.77176013	1	61	1.0334774	1
13	1.65636613	1	67	1.01041756	1
17	1.49953241	1	71	0.99667237	0
19	1.44288785	1	73	0.99021918	0
23	1.35496829	1	79	0.97231856	0
29	1.26169349	1	83	0.96145021	0
31	1.23719018	1	89	0.9465002	0
37	1.17656932	1	97	0.92869151	0
41	1.14404545	1		Total	28

The following figure represents the card models required in a deck for one player of the Gamification model. For processing numbers up to 70, the deck shall contain 28 cards as shown in the above table and illustrated in the following figure.

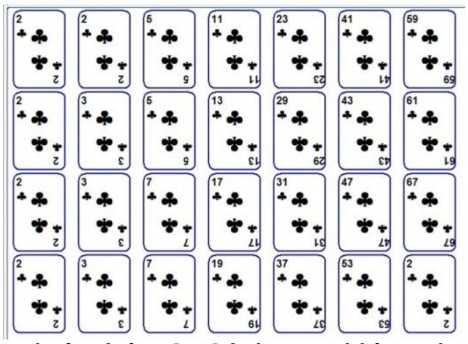


Figure 5 : Deck of cards for LCM Calculation Model for numbers up to 70

Optimisation 3 :: Determining the number of different suits required.

Unlike in the gaming model, the gamification model needs separate suites for each of the players. One can accommodate only up to four players using the four standard suites(♠ ♥ ♣ and ♦). When there are more than four players it becomes essential to introduce new suites with user defined cards.

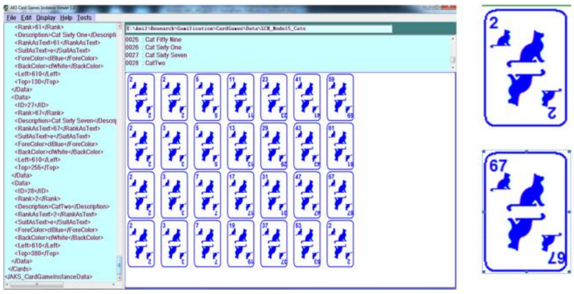


Figure 6 : Concepts of introducing user defined suites when number of player agents exceeds four.

Programmatic methods become essential to formulate such card models for presentation purposes as shown in the above screen shot.

Optimisation 4 :: Managing with limited Suits

However, as the LCM and HCF are Associative in nature, there is a possibility of applying a Divide and Conquer Strategy by which we can split the problem into sub problems (of two numbers each) and merge the result to get the final result, as explained above in case of Multi-Player gaming model of Gin Rummy. If such a policy is employed, we can process any number of inputs with just two distinct suites of cards. Such an implementation model will be more effective in a parallel or distributive computing system.

Optimisation 5 :: Finding Tactics for Shuffling Strategy

Another optimisation strategy which can be derived is the one in Shuffling strategy.., ie when some specific order for arranging the the Prime Numbers can influence the number of iterations required. All these optimisations depend on the nature of input numbers being provided for finding LCM or HCF . (In practical sense, the LCM or HCF Calculation will arise only as a sub problem of real world problem models in areas like cryptography, work distribution, profit sharing etc). Mathematical models alone will not be sufficient for handling such cases. This requires continuous analysis on the nature of input from particular environment of the actual problem model. Also the knowledge is to be updated with each of the learning epochs. So such systems need statistical models and reinforcement learning models for higher level optimisations.

E. Potential of Reinforcement Learning

In the case of LCM calculation, the above optimisation parameters can be mathematically calculated when the boundaries of the possible sample space are known. So, rather than keeping an unlimited deck of infinite number of cards, the deck could be limited to keep only 28 cards when the boundaries of input values got limited to [2..70] range.

However, in most of the real world situations, the boundaries may not be known at the beginning stages. Also there can be multiple factors which determine the optimisation parameters. For understanding the characteristic features of the sample space (boundaries for example) range and nature of the input and output values need to be studied. In such cases the statistical analysis of historical data can be helpful for formulating the strategies.

On the other hand, in adaptive learning systems, they should be intelligent enough to learn from experience and optimise their decisions based on the feedback from the environment. Such self learning systems shall keep a proper balance between exploration (for innovation) and exploitation (of acquired knowledge) while formulating the strategies aiming long term benefits and ultimate victory. In such systems, the conventional models in mathematics and statistics alone may not be sufficient enough to implement such process models.

Optimisation policies of such systems will get more and more powerful by enrichment of knowledge through experience of the Player Agent. Strategies and Policies are to be updated by accumulating the knowledge by going deeper into the dynamic sample space. In such cases, Reinforcement Learning will be the best method to formulate the Optimisation Strategies and to keep on updating them according to the changing needs of the environment.

VIII. Conclusions

Mathematical and statistical models which are used to represent Gaming models may become insufficient while handling certain instances of real world problems and their sub problems, especially those involving adaptive learning systems as in Gamification. Reinforcement Learning approach for experiencing the problem as well as the environment will be helpful in such situations, not only for understanding the tactics of the real game in an analytical manner, but also to represent those tactics as reusable meta-models. Based on these gaming and gamification meta-models, better solutions can be derived for industrial and real life problems.

IX. ACKNOWLEDGMENT

This research paper is an outcome of the analytical study and observations of the authors while experimenting with the theoretical concepts for their research projects.

X. REFERENCES

[1] Thomas Stahl and Markus Völter, "Model-Driven Software Development : Technology, Engineering, Management ," John Wiley & Sons Ltd, West Sussex, England ,2006

[2] Sebastian Deterding, Miguel Sicart, Lennart Nacke, Kenton O'Hara and Dan Dixon, "Gamification: Using Game Design Elements in Non-Gaming Contexts," ACM CHI '11 Extended Abstracts on Human Factors in Computing Systems

[3] Eric Freeman, Elisabeth Freeman, Bert Bates and Kathy Sierra, "Head First Design Patterns," O'REILLY, ISBN ,2004

[4] Leslie Pack Kaelbling, Michael L. Littman and Andrew W. Moore, "Reinforcement Learning: A Survey," Journal of Articial Intelligence Research, pp. 237-285 , 1996

[5] http://cardgames.io/ginrummy

[6] Gin Rummy - Mobile game from AI Factory in Google play store

[7] Roberto Battiti and Mauro Brunato, "The LION Way : Machine Learning plus Intelligent Optimization," LIONsolver Inc. Los Angels, 2013

[8] Richard S. Sutton and Andrew G. Barto, "Reinforcement Learning: An Introduction," The MIT Press, Cambridge, London, Second Edition, 2012

[9] http://www.tcs.com/resources/white_papers/Pages/Gamification-Insurance.aspx

[10] Gabriel Barata, Sandra Gama, Joaquim Jorge and Daniel Gonçalves, "Improving participation and learning with gamification," ACM Proceedings of the First International Conference on Gameful Design, Research, and Applications, pp. 10-17, 2013

[11] Jared Cechanowicz, Carl Gutwin, Briana Brownell and LarryGoodfellow, "Effects of Gamification on Participation and Data Quality in a Real-World Market Research Domain, " ACM Proceedings of the First International Conference on Gameful Design, Research, and Applications, pp. 58-65, 2013

[12] Thomas Juan and Leon Sterling, "A Meta-model for Intelligent Adaptive Multi-Agent Systems in Open Environments," Proceedings of the second international joint conference on Autonomous agents and multiagent systems, pp. 1024-1025,July 2003

[13] Michael Milton, "Head First Data Analysis," O'REILLY, ISBN 978-0-596-15393-9

AUTHOR PROFILES

Presently, Anil Kumar S is Final year M.Tech student in Software Engineering at Department of Computer Science, Cochin University of Science and Technology. His key areas of research interest are Multi disciplinary problems, Alternative solution models, Vedic Mathematics and Life Long Learning. At present, he is focusing on research in Software Engineering, Machine Learning and Reinforcement Learning in Gamification. The author can be contacted at aks.kerala.india@gmail.com

Muralidharan K B is working as Assistant Professor at Department of Computer Science, Cochin University of Science and Technology. He is having rich experience in Teaching as well as in Software Industry. His research interests include Massive dataset processing, Online algorithms for adaptive learning, Continuous stream algorithms and Virtualization. The author can be contacted at kbmuralidharan@gmail.com